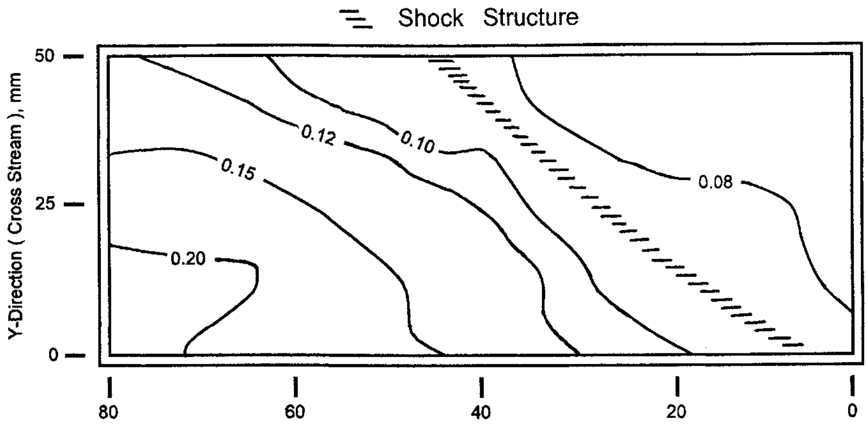

Fig. 3 Contours of constant hue showing the location of a shock at time $T + \Delta T$.

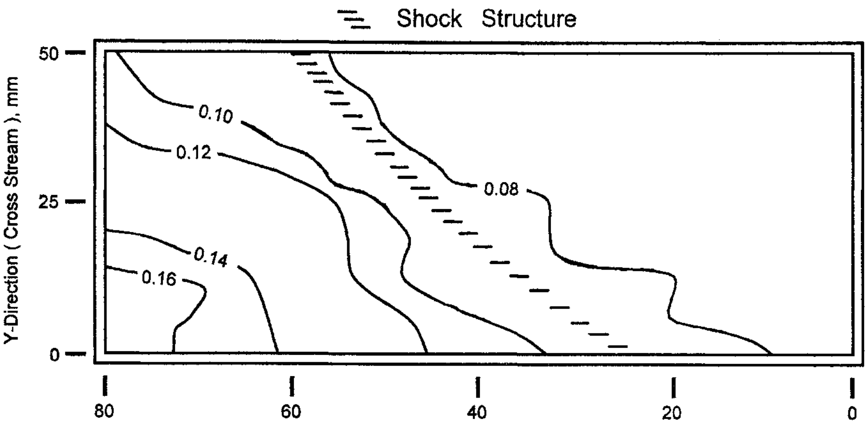

Fig. 4 Contours of constant hue showing the location of a shock line at time $T + 2\Delta T$.

Conclusions

In summary, a simple effective way for determining the starting transients in an intermittent supersonic wind tunnel has been shown. Liquid crystals of the Cholesteric type were used to observe the passing of the starting shock through the test section of a supersonic tunnel and included determination of its propagation velocity. This method of using liquid crystals may be applied and used both from a qualitative and quantitative viewpoint.

Acknowledgment

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Sensitivity of Flutter Response of a Wing to Shape and Modal Parameters

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Introduction

AUTOMATIC differentiation is emerging as a valuable tool for sensitivity calculations. ADIFOR, GRESS, PADRE-2, power calculus, and ODYSEE are some of the automatic differentiation

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packages[1] developed for differentiating Fortran77 codes. Automatic Differentiation of Fortran (ADIFOR) is a joint venture of Rice University and Argonne National Laboratory. ADIFOR processes a given Fortran code and generates a Fortran code for computing the derivatives of the desired output variables with respect to the independent variables by applying the chain rule of differentiation. Differentiating large codes by hand is cumbersome, divided differences are dependent on choice of step size, and symbolic programs may be infeasible for large codes. Automatic differentiation, on the other hand, can handle codes of arbitrary size and produce exact derivatives with no truncation error.

The problem of flutter instability was studied by Kapania and Issac[2] using a state-space aerodynamic representation and a Raleigh–Ritz structural formulation. The sensitivity of flutter speed to shape parameters of the wing were obtained analytically. In this Note, the flutter speeds for a wing are obtained using lifting-surface unsteady aerodynamics using modules from a system of programs called the flutter analysis system (FAST). Flutter speeds are obtained using a V–g type of solution. The derivatives of the flutter speed with respect to shape parameters of the wing are calculated using ADIFOR. To determine the importance of a particular mode to flutter, derivatives of the flutter speed with respect to natural frequencies, generalized mass, and generalized aerodynamic forces are computed using ADIFOR.

Flutter Calculations Using FAST

Flutter calculations are performed using FAST,[3] a system of programs based on the subsonic kernel function lifting-surface aerodynamic theory. The free vibration modes of the wing are fed into FAST, which solves the subsonic downwash integral equation and computes the generalized aerodynamic forces on the wing. The flutter speed of the wing is obtained using a V–g type of solution. The generalized aerodynamic forces are determined for a specific Mach number and for a range of values of the reduced frequency for the specified downwash distribution. These values are then interpolated to get aerodynamic forces for closely spaced values of the reduced frequency. The flutter equation solved by the program is

$$\left[\omega^2 - \omega_i^2(1+ig)\right]M_i q_i + \sum_{j=1}^{n} A_{ij} q_j = 0, \qquad i = 1, n \tag{1}$$

where ω is the vibration frequency, ω_i is the frequency of the ith natural vibration mode, M_i is the generalized mass associated with the ith natural vibration mode, g is the incremental damping, A_{ij} are the generalized aerodynamic forces resulting from the pressure induced by the jth mode acting through the displacements of the ith mode and q_i is the ith component of the flutter eigenvector.

In terms of the nondimensional generalized aerodynamic forces $\bar{A}_{ij}$, the Eq. (1) can be written as an eigenvalue problem in the form

$$\sum_{j=1}^{n} (C_{ij} - \delta_{ij}\Omega) q_j = 0, \qquad i = 1, n \tag{2}$$

where the eigenvalue Ω is given by

$$\Omega = (\omega_0 b_0 / V)^2 (1 + ig) \tag{3}$$

and

$$C_{ij} = \left(\frac{\omega_0}{\omega_i}\right)^2 \left[\frac{\rho b_0^3 \bar{A}_{ij}}{2M_i} + \delta_{ij} k^2\right] \tag{4}$$

where ω_0 is a reference frequency, b_0 the reference length (usually root semichord), k the reduced frequency, ρ the air density and V the airspeed.

This eigenvalue problem is solved for a range of values of the reduced frequency and monitored for crossings on the V–g plane when the incremental damping g goes to zero. At each of these crossings, the values of the airspeed and the frequency are noted. The critical flutter speed is the lowest speed at which the damping of the structure goes to zero.

Sensitivity Derivatives Using ADIFOR

For all of the results presented, the wing shown in Fig. 2 of Ref. 2 is used with the wing skins made of 0-deg laminated graphite/epoxy (T300/N5208) with the following material properties: $E_1 = 181 \times 10^9$ Pa, $E_2 = 10.3 \times 10^9$ Pa, $\nu_{12} = 0.28$, $G_{12} = 7.17 \times 10^9$ Pa, and $\rho = 1600$ kg/m^3.

The sensitivity derivatives of the flutter speed with respect to shape parameters have been calculated using ADIFOR. The sensitivity results with respect to wing area and sweep angle and the results from reanalysis are shown in Figs. 1 and 2 for the wing at $M = 0.6$. By performing one sensitivity calculation at the baseline, this method gives a linear approximation to the flutter speeds of the wing for changes in the wing shape parameters about the baseline. This information is useful for preliminary design purposes, as it avoids the necessity of a reanalysis for small changes in any of the shape parameters.

Complex wing structures are often modeled with a large number of degrees of freedom, and a vibration analysis yields a large number of free vibration modes. A certain number of these modes have to be fed into FAST to generate the generalized aerodynamic forces required for flutter analysis. Some of these modes do not actively participate in the flutter, and using several modes to determine flutter instability leads to larger computational cost. In situations where the natural frequencies and mode shapes of the wing are measured from experiments, one has large amount of modal data and has to determine the number of modes that are required for reasonably estimating the flutter speed. One could make a judicious choice of the number of modes that are required for flutter analysis based on the sensitivity derivatives of the flutter speed with respect to the modal parameters of the wing.

The sensitivity of flutter speed to modal parameters were obtained using ADIFOR. The derivatives of the flutter speed with respect to natural frequencies of the wing are shown in Fig. 3. Nine modes were used for the flutter analysis. It can be seen that if the third natural frequency is increased by 1 rad/s, then the flutter speed increases by 1.5 m/s. It is also seen that higher modes do not contribute much to the flutter. The derivatives of the flutter speed with respect to

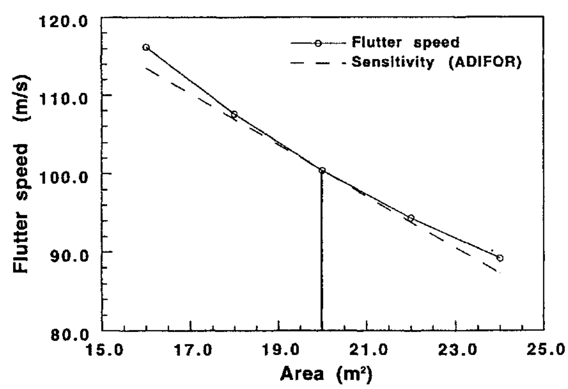

Fig. 1 Flutter speed vs area, *M* = 0.6, from FAST: *AR* = 10, area = 20 m², *TR* = 0.5, sweep = 30 deg.

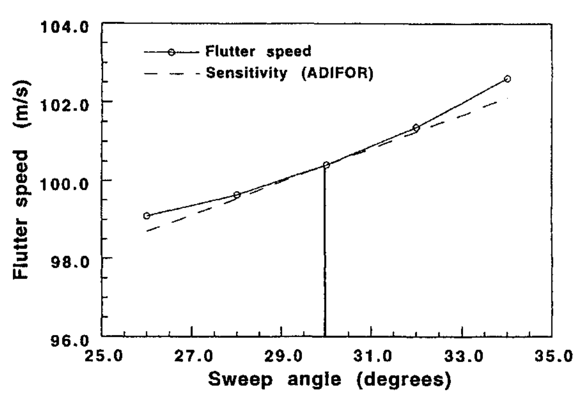

Fig. 2 Flutter speed vs sweep angle, *M* = 0.6, from FAST: *AR* = 10, area = 20 m², *TR* = 0.5, sweep = 30 deg.

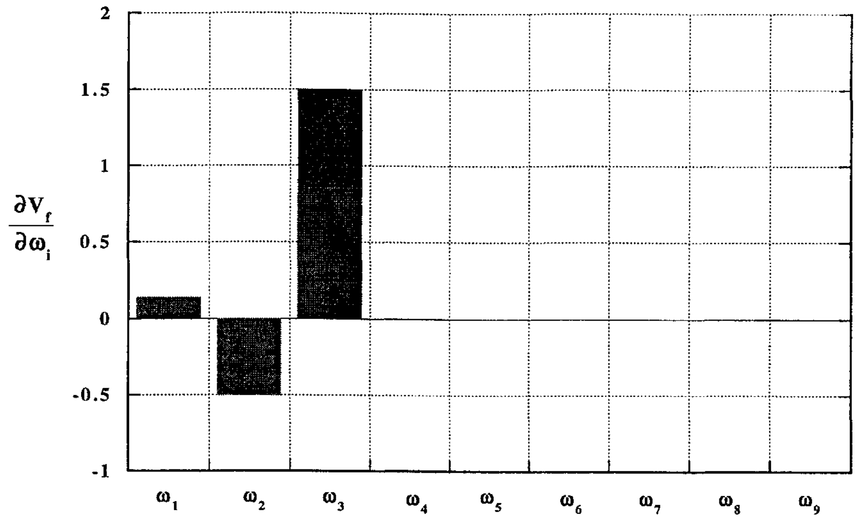

Fig. 3 Flutter speed sensitivity to natural frequency, M = 0.6: AR = 10, area = 20 m^2, TR = 0.5, sweep = 30 deg.

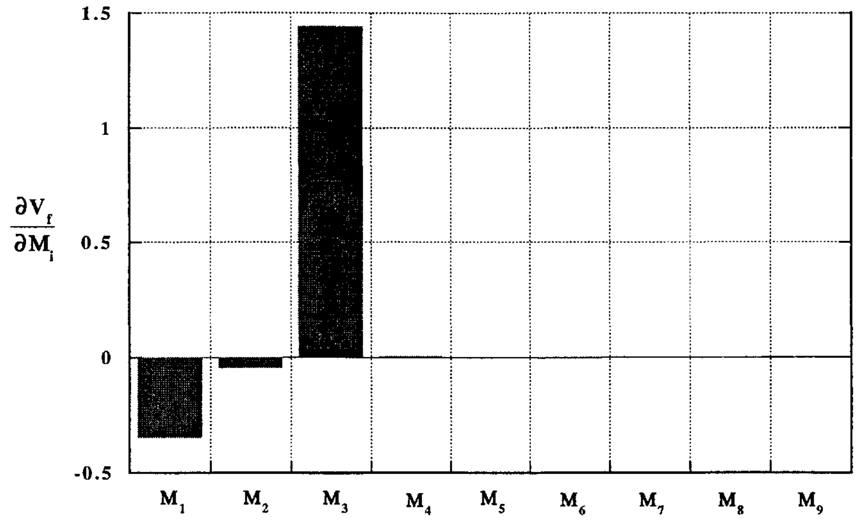

Fig. 4 Flutter speed sensitivity to generalized mass, M = 0.6: AR = 10, area = 20 m^2, TR = 0.5, sweep = 30 deg.

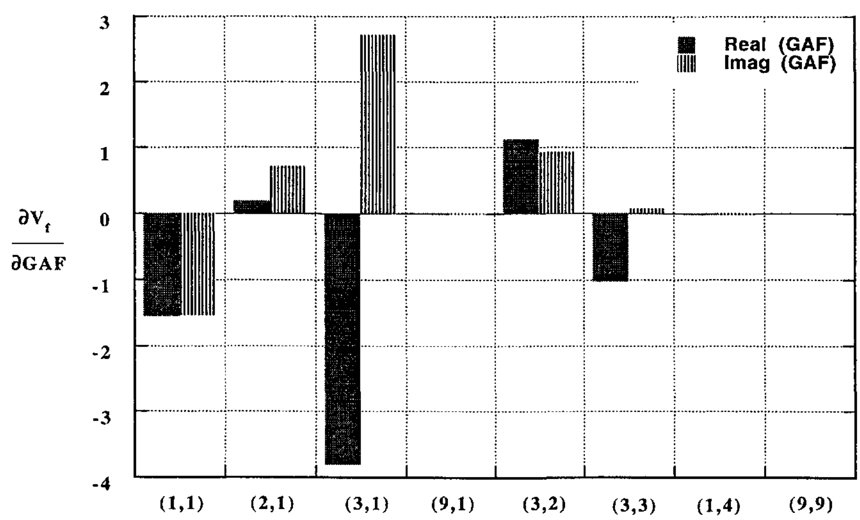

Fig. 5 Flutter speed sensitivity to generalized aerodynamic forces, M = 0.6: AR = 10, area = 20 m^2, TR = 0.5, sweep = 30 deg.

generalized mass is given in Fig. 4, and the flutter speed sensitivity to the real and imaginary parts of the generalized aerodynamic forces (GAF) is given in Fig. 5. The (i, j) term in Fig. 5 stands for the nondimensional generalized aerodynamic force resulting from the pressure induced by the jth mode acting through the displacements of the ith mode. The sensitivities of flutter speed with respect to modal parameters give an estimate of the importance of a particular mode to flutter.

Concluding Remarks

Flutter speed and sensitivity calculations are performed using a lifting-surface unsteady aerodynamic theory using the generalized aerodynamic forces from a system of programs called FAST and a V–g type of solution. The sensitivity of flutter speed to shape and modal parameters have been obtained using ADIFOR. The shape derivatives of the flutter response of a wing would be very useful to a designer in the initial design phase, thus avoiding the necessity of a reanalysis for small changes in the design parameters. The sensitivity derivatives of the flutter response to modal parameters are useful for identifying the modes that are important aeroelastically.

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Thermally Induced Vibration of a Symmetric Cross-Ply Plate with Hygrothermal Effects

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Introduction

THE response of filamentary composite plates to thermally induced vibrations when exposed to a hygrothermal environment is of major concern. The effects of heating on the dynamic response of plates were investigated as early as 1953 when Tsien[1] derived the general equations of motion for a heated, homogeneous, isotropic plate. Years later, Kao and Pao[2] developed the governing equations for a simply supported heterogeneous, anisotropic plate with rapid heating on one side. In the early 1970s, Whitney and Ashton[3] added expansional strains to the laminated plate equations. Over the next two decades, the effects of moisture on plate response have been studied using various models and techniques.[4–6] The present work studies the vibration of symmetric cross-ply plates under unsteady temperature effects and an unsteady moisture environment for the simply supported case.

Governing Equations

From classical laminated plate theory the transverse motion of a thin, symmetric, cross-ply plate exposed to elevated temperature and moisture conditions is

$$D_{11}w_{,xxxx} + 2(D_{12} + 2D_{66})w_{,xxyy} + D_{22}w_{,yyyy} + \rho h w_{,tt} = -M^T_{x,xx} - M^m_{x,xx} - M^T_{y,yy} - M^m_{y,yy} \tag{1}$$

in which ρ denotes the mass density, h is the thickness, D_{ij} are the laminate bending stiffnesses, and M_x^T, M_y^T, $M^m_{,x}$, and M_y^m represent the thermal and hygrothermal moments and are defined by

$$\begin{aligned}
\left(M_x^T, M_y^T\right) &= \sum_{k=1}^{R}\left[\left(\gamma_1^\alpha\right)_k, \left(\gamma_2^\alpha\right)_k\right]\int_{h_{k-1}}^{h_k} \Delta T z\,\mathrm{d}z \\
\left(\gamma_1^\alpha\right)_k &= (\bar{Q}_{i1}\alpha_x + \bar{Q}_{i2}\alpha_y)_k \\
\left(M_x^m, M_y^m\right) &= \sum_{k=1}^{R}\left[\left(\gamma_1^\beta\right)_k, \left(\gamma_2^\beta\right)_k\right]\int_{h_{k-1}}^{h_k} \Delta m z\,\mathrm{d}z \\
\left(\gamma_i^\beta\right)_k &= (\bar{Q}_{i1}\beta_x + \bar{Q}_{i2}\beta_y)_k
\end{aligned} \tag{2}$$

The thermal excitation considered is a suddenly applied heat flux[7]

$$\begin{aligned}
\Delta T = \frac{hq_0}{k_z}\Bigg[\frac{\beta^* t}{\pi^2} + \frac{1}{2}\left(\frac{z}{h} + \frac{1}{2}\right)^2 \\
- \frac{1}{6} - \frac{2}{\pi^2}\sum_{j=1}^{\infty}\frac{(-1)^j}{j^2}e^{-j^2\beta t}\cos j\pi\left(\frac{z}{h} + \frac{1}{2}\right)\Bigg]
\end{aligned} \tag{3}$$

$$\beta* = \frac{\pi^2\alpha}{h^2}$$

The hygrothermal excitation used is characterized by a sudden elevated moisture level on the upper plate surface

$$\begin{aligned}
\Delta m &= \delta_u + \sum_{r=0}^{\infty} B_r e^{-\lambda_r^2\beta t}\cos\lambda_r\left(z + \frac{h}{2}\right) \\
B_r &= \frac{2}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} \delta(z)\cos\lambda_r\left(z + \frac{h}{2}\right)\mathrm{d}z - \frac{4\delta_u(-1)^r}{(2r+1)\pi} \\
\delta_u &= m_u - m_0, \qquad \delta(z) = m(z) - m_0
\end{aligned} \tag{4}$$

Solution

An exact solution will be derived for a plate with all sides simply supported by assuming a deflection of the form

$$\begin{aligned}
w &= w_1(x, y, t) + w_2(x, y, t) \\
&= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(K_{mn} + q_{mn})\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}
\end{aligned} \tag{5}$$

Here w_1 (or K_{mn}) is the solution of Eq. (1) neglecting the inertia term and w_2 (or q_{mn}) is the solution of

$$D_{11}w_{2,xxxx} + 2(D_{12} + 2D_{66})w_{2,xxyy} + D_{22}w_{2,yyyy} + \rho h w_{2,tt} = -\rho h w_{1,tt} \tag{6}$$

Letting $a_m = m\Pi/a$ and $b_n = n\Pi/b$ the thermal and hygrothermal moments are expressed as

$$\begin{aligned}
&\left(M_x^T, M_y^T, M_x^m, M_y^m\right) \\
&\quad = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(B_{mn}, C_{mn}, D_{mn}, E_{mn})\sin a_m x \sin b_n y \\
&(B_{mn}, C_{mn}) = \frac{4(\cos m\pi - 1)(\cos n\pi - 1)}{mn\pi^2} \\
&\quad \times \sum_{k=1}^{R}\left[\left(\gamma_1^\alpha\right)_k, \left(\gamma_2^\alpha\right)_k\right]\int_{h_{k-1}}^{h_k}\Delta T z\,\mathrm{d}z \\
&(D_{mn}, E_{mn}) = \frac{4(\cos m\pi - 1)(\cos n\pi - 1)}{mn\pi^2} \\
&\quad \times \sum_{k=1}^{R}\left[\left(\gamma_1^\beta\right)_k, \left(\gamma_2^\beta\right)_k\right]\int_{h_{k-1}}^{h_k}\Delta m z\,\mathrm{d}zv
\end{aligned} \tag{7}$$

The total response is then

$$K_{mn} = \left(\frac{1}{G_{mn}}\right)\left(B_{mn}a_m^2 + C_{mn}b_n^2 D_{mn}a_m^2 E_{mn}b_n^2\right)$$

$$G_{mn} = \left[D_{11}a_m^4 + 2(D_{12} + 2D_{66})a_m^2 b_n^2 + D_{22}b_n^4\right]$$

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